

In lecture 3 we introduced several acceleration curves: constant acceleration, simple harmonic, modified trapezoidal, modified sine, and cycloidal. All of these acceleration functions can be defined by the same set of equations with only a change of parameters. This is referred to as the SCCA (Sine - constant - cosine Acceleration) acceleration functions and will all have the same general shape

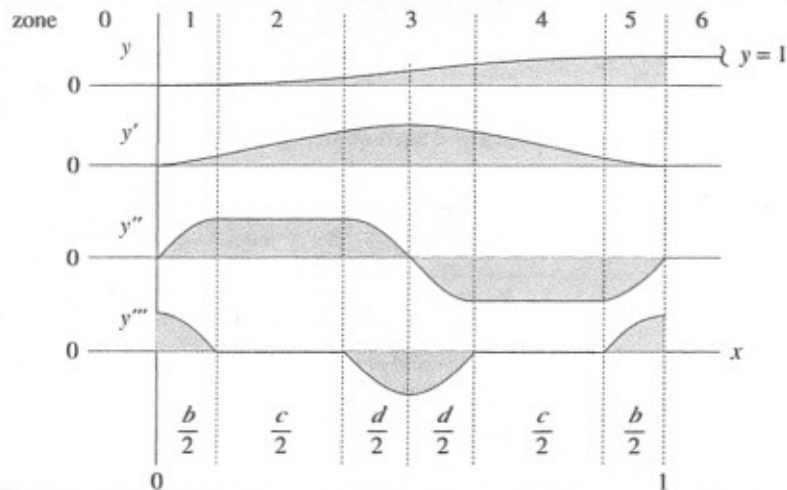


FIGURE 8-17 Parameters for the normalized SCCA family of curves

For each zone there will be a set of equations for $s, v, a,$ and j that is defined by the associated coefficients

TABLE 8-2 Parameters and Coefficients for the SCCA Family of Functions

Function	b	c	d	C_v	C_a	C_j
constant acceleration	0.00	1.00	0.00	2.0000	4.0000	infinite
modified trapezoid	0.25	0.50	0.25	2.0000	4.8881	61.426
simple harmonic	0.00	0.00	1.00	1.5708	4.9348	infinite
modified sine	0.25	0.00	0.75	1.7596	5.5280	69.466
cycloidal displacement	0.50	0.00	0.50	2.0000	6.2832	39.478

$y = \frac{s}{h}$
normalized displacement

For example, in zone 1

$$y = C_a \left[\frac{b}{\pi} x - \left(\frac{b}{\pi}\right)^2 \sin\left(\frac{\pi x}{b}\right) \right] \quad \frac{dy}{dx} = C_a \left[\frac{b}{\pi} - \frac{b}{\pi} \cos\left(\frac{\pi}{b} x\right) \right]$$

$$\frac{d^2y}{dx^2} = C_a \sin\left(\frac{\pi}{b} x\right) \quad \frac{d^3y}{dx^3} = C_a \frac{\pi}{b} \cos\left(\frac{\pi}{b} x\right)$$

- For $s, v, a,$ and j equations for zones 2 through 6 see p.399 to 401 Eq. 8-15 to 8-20. of Norton's text 4-2
- To apply the SCCA functions to an actual cam

$$\begin{aligned} \text{Displacement} &= s = h y && (\text{length}) \\ \text{Velocity} &= v = \frac{h}{\beta} \left(\frac{dy}{dx} \right) \omega && (\text{length/s}) \\ \text{Acceleration} &= a = \frac{h}{\beta^2} \left(\frac{d^2y}{dx^2} \right) \omega^2 && (\text{length/s}^2) \\ \text{Jerk} &= j = \frac{h}{\beta^3} \left(\frac{d^3y}{dx^3} \right) \omega^3 && (\text{length/s}^3) \end{aligned}$$

For the double-dwell problem we introduced on pp. 3-2

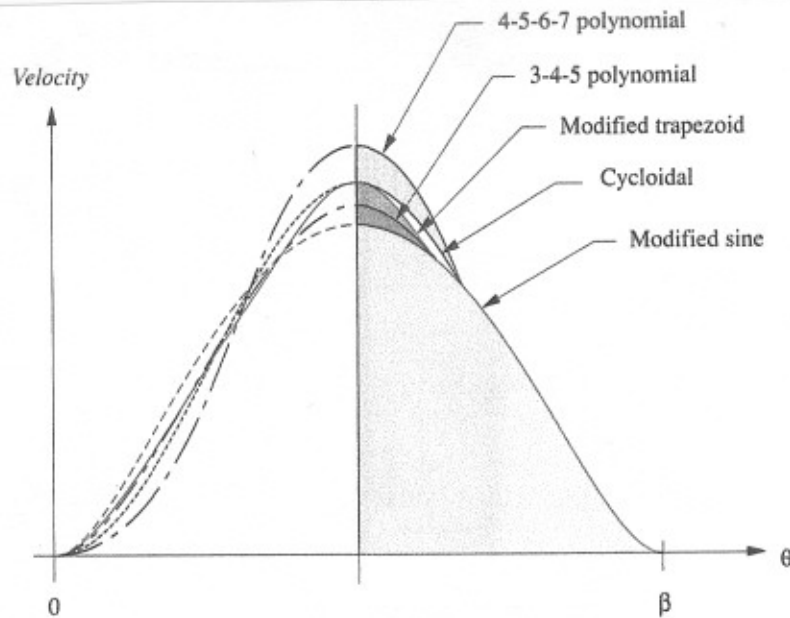


FIGURE 8-21
Comparison of five double-dwell cam velocity functions

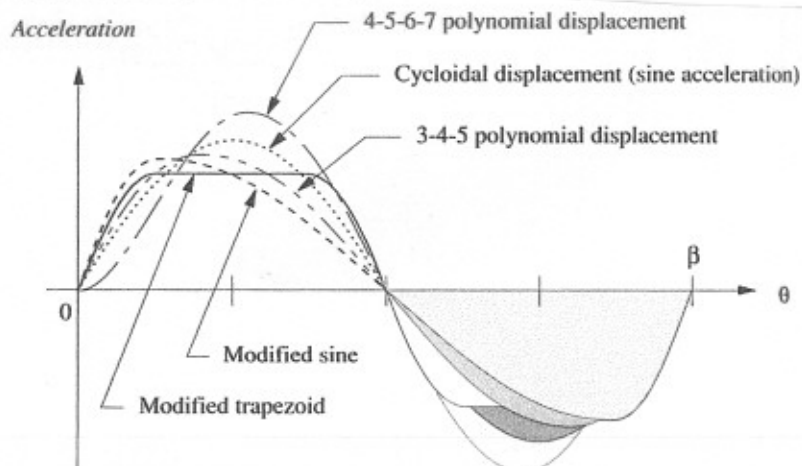


FIGURE 8-19
Comparison of five double-dwell cam acceleration functions

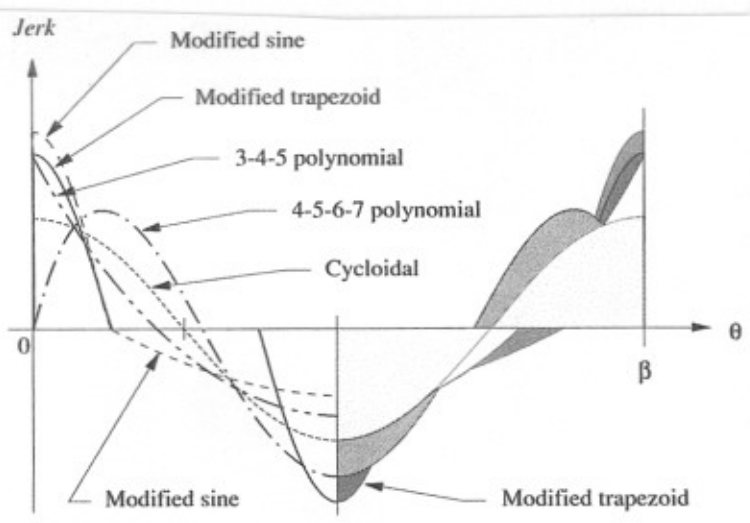
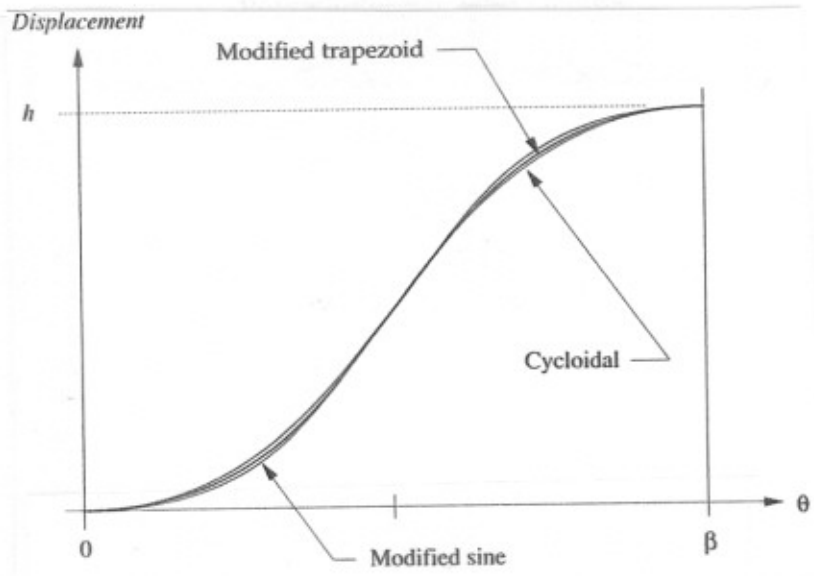


FIGURE 8-20 Comparison of five double-dwell cam jerk functions

TABLE 8-3 Factors for Peak Velocity and Acceleration of Some Cam Functions

Function	Max. Veloc.	Max. Accel.	Max. Jerk	Comments
Constant accel.	$2.000 h/\beta$	$4.000 h/\beta^2$	Infinite	∞ jerk—not acceptable
Harmonic disp.	$1.571 h/\beta$	$4.945 h/\beta^2$	Infinite	∞ jerk—not acceptable
Trapezoid accel.	$2.000 h/\beta$	$5.300 h/\beta^2$	$44 h/\beta^3$	Not as good as mod. trap.
Mod. trap. accel.	$2.000 h/\beta$	$4.888 h/\beta^2$	$61 h/\beta^3$	Low accel. but rough jerk
Mod. sine accel.	$1.760 h/\beta$	$5.528 h/\beta^2$	$69 h/\beta^3$	Low veloc., good accel
3-4-5 poly. disp.	$1.875 h/\beta$	$5.777 h/\beta^2$	$60 h/\beta^3$	Good compromise
Cycloidal disp.	$2.000 h/\beta$	$6.283 h/\beta^2$	$40 h/\beta^3$	Smooth accel. and jerk.
4-5-6-7 poly. disp.	$2.188 h/\beta$	$7.526 h/\beta^2$	$52 h/\beta^3$	Smooth jerk, high accel.

Different acceleration functions will provide different dynamic characteristics. For low acceleration choose modified trapezoidal. For low velocity choose modified sine. The designer must ultimately choose the appropriate function. Notice how similar the displacement curves look for the double-dwell problem.



Polynomial Functions

Kloomok and Muffley developed a system of CAM Design that uses three analytical functions a) cycloid; b) harmonic; c) eighth-power polynomial. The selection of the profiles to suit particular requirements is made according to the following criteria:

- ① The cycloid provides zero acceleration at both ends. Therefore it can be coupled to a dwell at each end. Because the pressure angle is relatively high and the acceleration returns to zero two cycloids should not be coupled together.
- ② The harmonic provides the lowest peak acceleration and pressure angle of the three curves.
- ③ The eighth - power polynomial has a non-symmetrical acceleration curve and provides a peak acceleration and pressure angle intermediate between the harmonic and the cycloid.

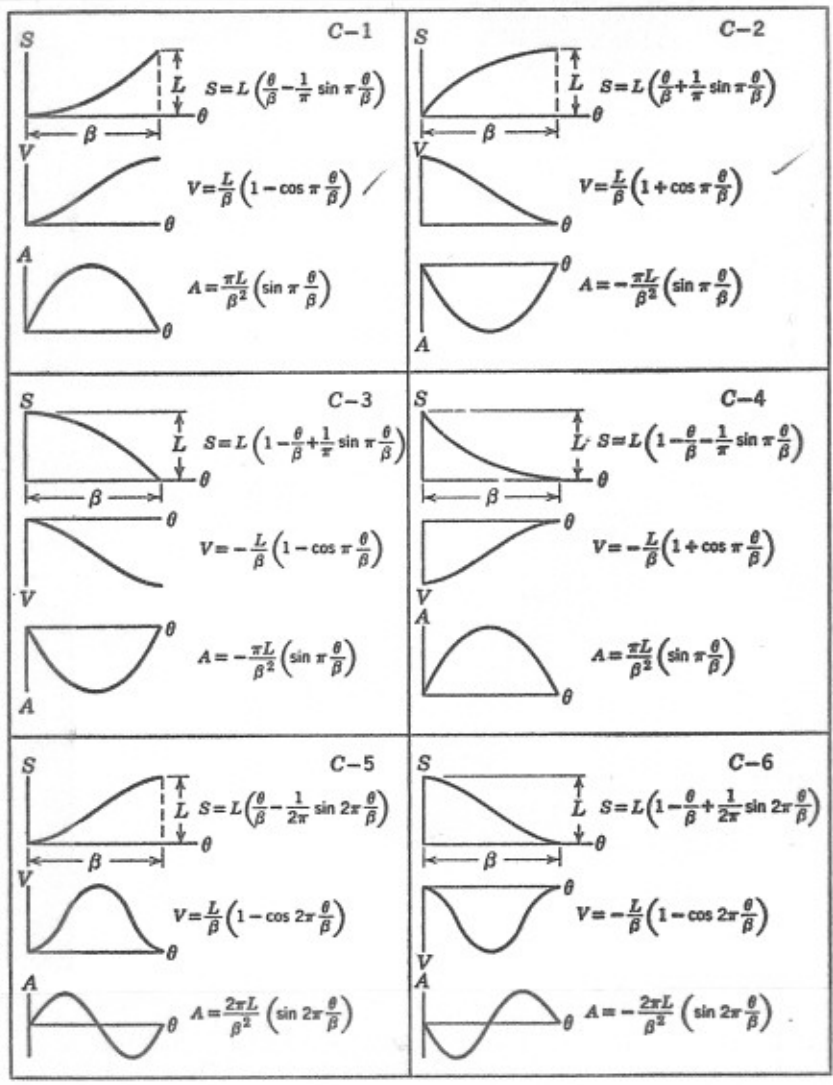


FIGURE 3.15 Cycloidal motion characteristics: S = displacement, inches; V = velocity, inches per degree; A = acceleration, inches per degree squared. (M. Kloomok and R. V. Muffley, "Plate Cam Design—with Emphasis on Dynamic Effects," *Prod. Eng.*, February 1955.) N. B. For SI units, S = displacement, millimeters; V = velocity, millimeters per degree; A = acceleration, millimeters per degree squared.

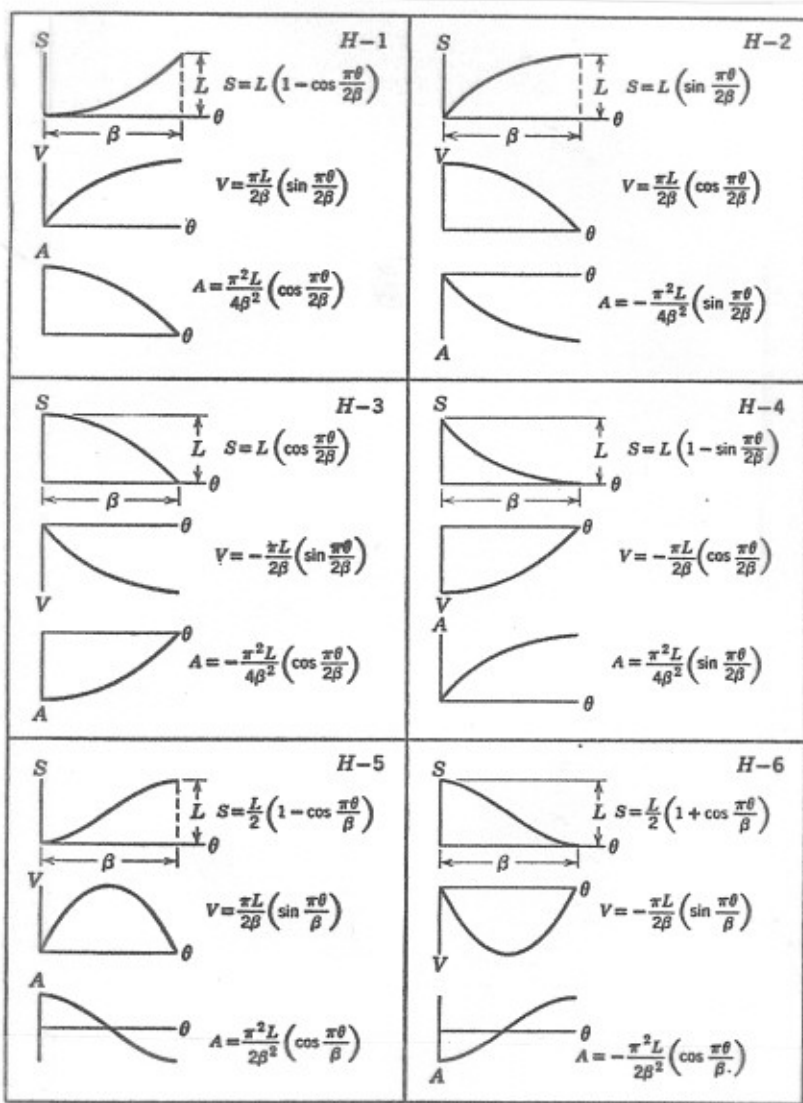


FIGURE 3.16 Harmonic motion characteristics: S = displacement, inches; V = velocity, inches per degree; A = acceleration, inches per degree squared. (M. Klooomok and R. V. Muffley, "Plate Cam Design—with Emphasis on Dynamic Effects," *Prod. Eng.*, February 1955.) *N. B.* For SI units, S = displacement, millimeters; V = velocity, millimeters per degree; A = acceleration, millimeters per degree squared.

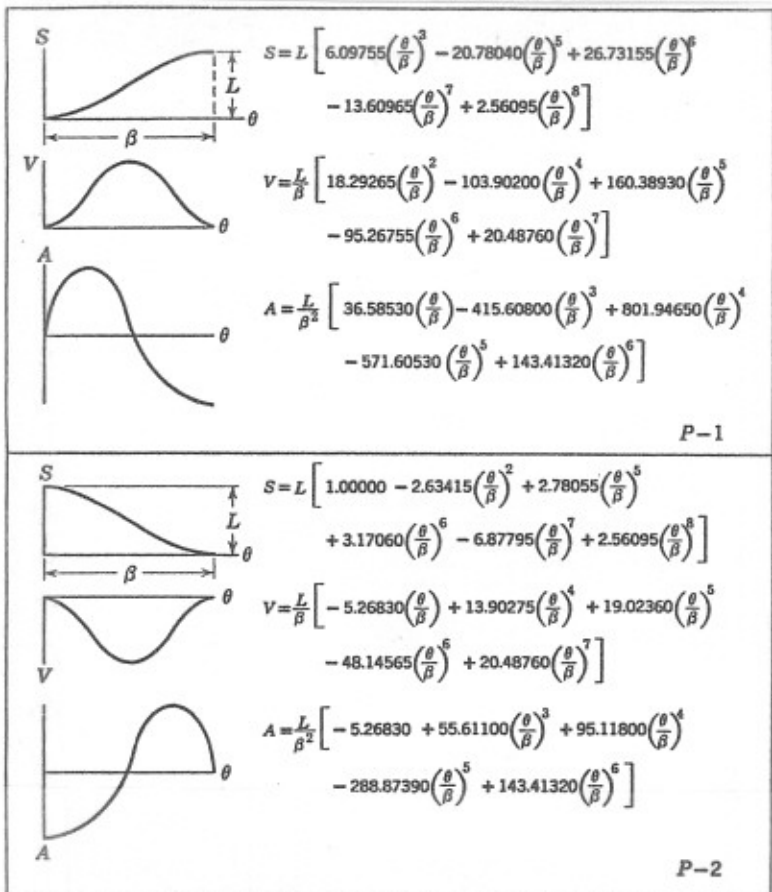
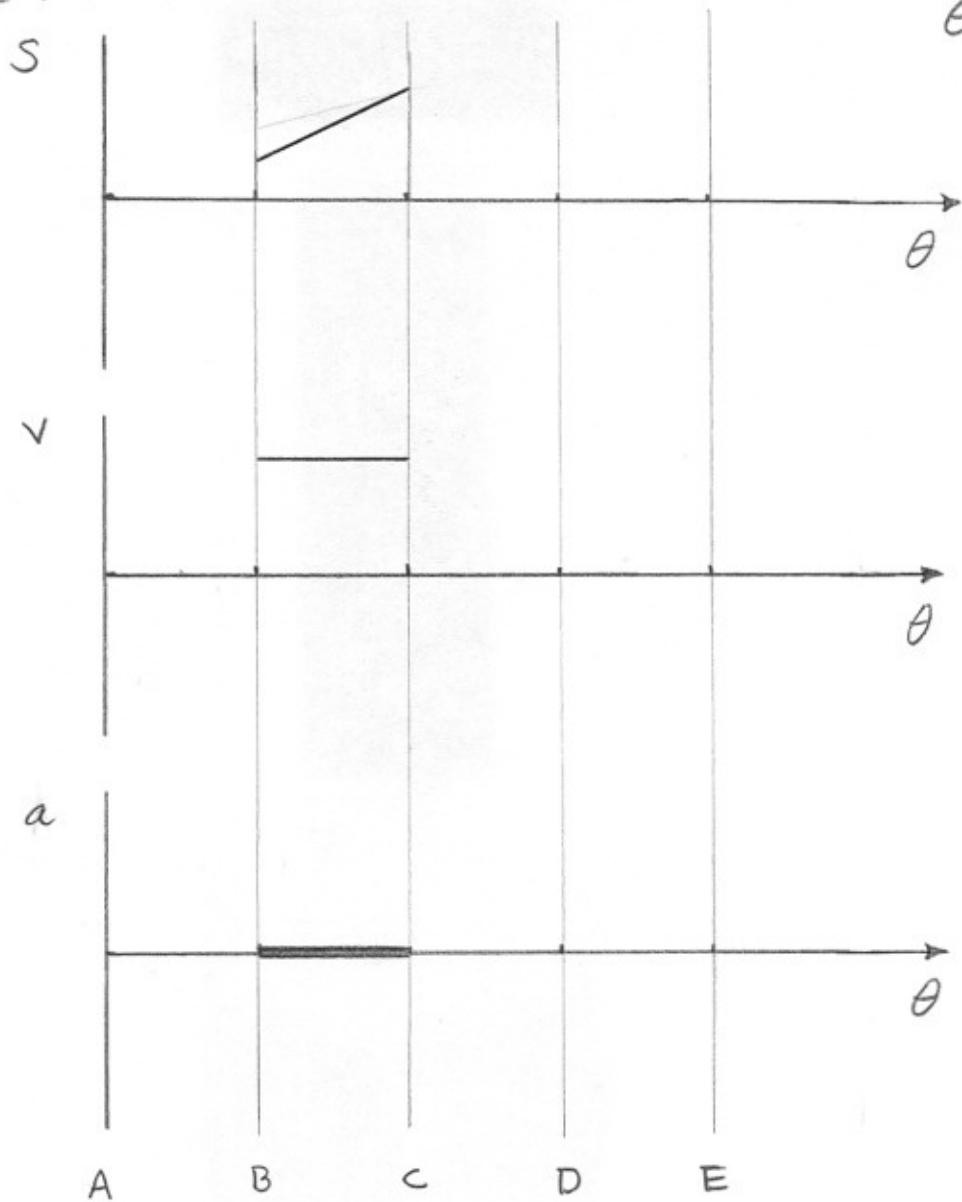


FIGURE 3.17 Eighth-power polynomial motion characteristics: S = displacement, inches; V = velocity, inches per degree; A = acceleration, inches per degree squared. (M. Klooomok and R. V. Muffley, "Plate Cam Design—with Emphasis on Dynamic Effects," *Prod. Eng.*, February 1955.) *N. B.* For SI units, S = displacements, millimeters; V = velocity, millimeters per degree; A = acceleration, millimeters per degree squared.

From Mechanisms and Dynamics of Machinery (Mabie and Reinholtz)

Example A roller follower is to move through a total displacement and return with no dwells in the cycle. 4-6
 Because of the operation performed by the mechanism, a portion of the outward motion must be at constant velocity.
 Determine the motion curves to be used if $s = v = a = 0$ at $\theta = 0$



- From A to B the curve to use is _____
 From B to C the curve to use is constant velocity
 From C to D the curve to use is _____
 From D to E the curve to use is _____